The errors associated with the network models are represented by the normalized root mean square (nrms) error and the maximum percent error. The nrms error is the typical rms normalized using the sum of squares of the desired outputs. The error statistics reported represent the average statistics for 10 networks trained with different weight initializations. Table 1 contains the error statistics for Quickprop trained with the full training set and the compressed training sets. Table 2 contains the error statistics for the LVQ estimator using full and compressed training sets. The first four rows in each table are for the training data.

Training times are shown in Table 3. The first two columns show the training times for LVQ and Quickprop. The last column indicates the total training time, that is, the time to reduce the training set added to the time for Quickprop training using the reduced training sets.

Conclusion

In this investigation, the issue of reducing the training time of a feedforward neural network with backpropagation was addressed. To improve training efficiency, the networks were trained on a reduced data set. The data reduction was obtained using the LVQ algorithm. Various compression ratios were investigated to compress the training data. The impact of various compression ratios on training was analyzed through the comparison of error statistics on the validation data.

It was demonstrated that the compressed data set could be used to train other networks or as the codebook set for the LVQ estimator. It should be noted that the LVQ, when used as a neural network model, can provide much faster estimation. However, the output of the LVQ estimator was noisy with larger errors. These errors are primarily because of artifacts introduced by the formation of the codebook vectors. If an acceptable interval of errors in estimation is predefined, the noise in the output of the LVQ estimator can be controlled by designing an appropriate number of codebook vectors. Thus, if the estimation time is of primary concern, the LVQ neural network model could be used as an estimator.

It can be concluded from the presented investigation that the training efficiency of the feedforward neural network model can be improved by reducing the training data using the LVQ algorithm. As training data was reduced, there was a comparatively small increase in the errors, whereas the training times were decreased significantly. It should be noted that such an improvement in efficiency by reducing the training time may be very useful for very large data sets, such as the data compiled during the SSME mainstage operation.

Acknowledgments

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References

[1]Bickmore, T., "Probabilistic Approach to Sensor Data Validation," AIAA Paper 92-3163, July 1992.

[2]Makel, D. K., Flaspohler, W. H., and Bickmore, T. W., "Sensor Data Validation and Reconstruction, Phase 1: System Architecture Study," NASA CR 187122, 1991.

[3]Chen, S., Billings, S. A., and Grant, P. M., "Non-linear Systems Identification Using Neural Networks," Univ. of Edinburgh, Research Rept. 370, Edinburgh, Scotland, UK, Aug. 1989.

[4]Meyer, C. M., and Maul, W. A., "The Application of Neural Networks to the SSME Startup Transient," AIAA Paper 91-2530, June 1991.

[5]Kohonen, T., *Self-Organization and Associative Memory*, Springer-Verlag, Berlin, 1989, pp. 119–157, 199–202.

[6]Fahlman, S. E., "An Empirical Study of Learning Speed in Back-Propagation Networks," Carnegie Mellon Univ., Technical Rept. CMU-CS-88-162, Pittsburgh, PA, Sept. 1988.

[7]Rumelhart D. E., Hinton, G. E., and Williams, R. J., "Learning Internal Representations by Error Propagation," *Parallel Distributed Processing Explorations in the Microstructure of Cognition*, edited by D. E. Rumelhart, J. L. McClelland, and the PDP Research Group, Vol. 1: Foundations, MIT Press, Cambridge, MA, 1986, pp. 318–364.

[8]Szewczyk, Z., and Hajela, P., "Neurocomputing Strategies In Decomposition Structural Design," *AIAA/ASME/ASCE/AHS/ASC 34th Structures, Structural Dynamics, and Materials Conference, AIAA/ASME Adaptive Structures Forum*, Vol. 4, AIAA, Washington, DC, 1993, pp. 2458–2465.

Simplified Analysis of General Instability of Stiffened Shells with Cutouts in Pure Bending

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Introduction

FAILURE of stiffened shells in a general instability mode occurs when the spring constant of the transverse stiffening frames is reduced below a minimum necessary value for the confinement of instability failure between adjacent frames. Under such conditions, general instability failure can be spread over a number of frame spacings in a waveform shape described as an "inward bulge" occurring at the extreme compression fibers. A comprehensive discussion of the complex problem of general instability of stiffened shells in bending, pressure, torsion, transverse shear, and combined bending and torsion has been reported by Becker.[1] His formulation of the problem, based on a correlation of analytical and experimental results obtained prior to 1958, has become the backbone of most of industry's standard design manuals, e.g., Bruhn.[2] Furthermore, a number of analytical and experimental studies on the subject of general instability of stiffened or unstiffened shells with or without cutouts are available.[3,4] For design purposes, Shanley[5] has proposed a simple criterion which is based on a correlation of analytical results obtained from a simplified model and experimental data. Similar criteria have also been proposed by Gerard[6] and Hoff.[7] However, one should be aware of the fact that all of these general instability criteria are based on tests which may not be representative of contemporary fuselage designs.[8,9] Despite the criticism, Shanley's criterion remains in general use due to its simplicity, its direct correlation to experimental data, and the relatively few and easily obtainable factors involved in the calculations. However, one distinct limitation of the aforementioned criteria is that they are not directly applicable to incomplete frames at locations where substantial fuselage cutouts, such as cargo doors, are present. In this work, a simple analytical procedure is presented which may be useful in checking the design of the stiffening frames of cylindrical fuselages, with or without cutouts, for failure by general instability under bending conditions. The proposed formulation traces the basic steps of Shanley's original work for complete circular frames and extends it to incomplete frames, with or without edge stiffeners at the perimeter of the cutout.

Formulation

Simplified Model

The basic model used in Ref. 5 for general instability of a fuselage structure is a spring-bar assembly such as the one shown in Fig. 1. The horizontal bars represent the sheet-stringer elements

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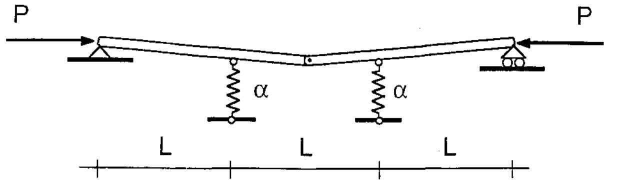

Fig. 1 Simplified model for general instability failure of stiffened shells.

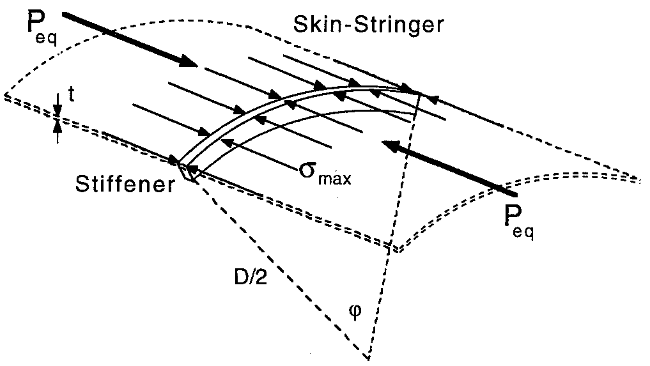

Fig. 2 Equivalent axial load due to bending stresses along an arc.

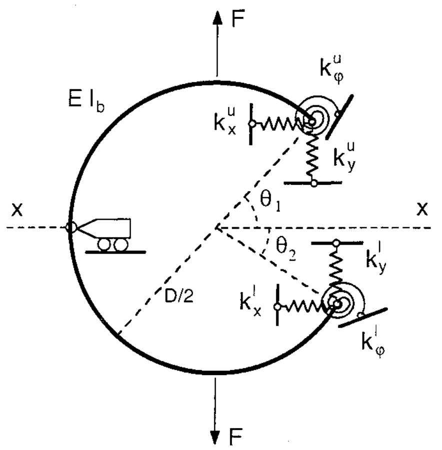

Fig. 3 Structural model for a circular frame with edge reinforcement.

which, at the vicinity of the most stressed point of the shell structure, approach their critical load as columns and tend to deform as if a hinge connection had formed between them. The spring elements resemble the lateral stiffness of the system provided by transverse frames. The general instability failure mode implied by this simple model is supported by experimental evidence.[10] The spring constant α may be expressed as

$$\alpha = K_1 (P/L) \tag{1}$$

where, for this particular example, $K_1 = 1.5$, but the actual value of the constant K_1 is immaterial in subsequent calculations. In general, Eq. (1) may be viewed as representing a number of spring-bar assemblies by merely adjusting the coefficient K_1, e.g., see Timoshenko and Gere.[11]

Equivalent Load on the Circumference of a Circular Transverse Frame

Assuming a fairly uniform distribution of sheet-stringer properties in the vicinity of the extreme fiber, the equivalent normal force P_{eq} over an arc φ, as shown in Fig. 2, may be written as

$$P_{eq} = K_2 (M/D) \tag{2}$$

where

$$K_2 = (D^3 t/4I)\varphi \tag{3}$$

M is the applied bending moment, D is the diameter of the shell, and t and I are the equivalent thickness and moment of inertia of the skin-stringer system, respectively.

Bending Stiffness of a Circular Frame

The radial bending stiffness α of a complete or incomplete circular frame can be obtained from the structural configuration and loads shown in Fig. 3. The added springs represent structural stiffeners usually used for reinforcement at the top and bottom edges of a fuselage cutout. The pair of equal but opposite forces arranged along the vertical diameter represents, at a unit elongation of the vertical diameter, the numerical value of the required radial stiffness. In the general case, asymmetric patterns of displacements will result, which are not consistent with the constraints provided by other parts of the fuselage on either side of the section with the cutout. For this reason, some additional support may be included in the structural model, consistent with these constraints. In most practical cases, a simple vertical support diametrically opposite to the cutout, as shown in Fig. 3, will suffice to provide displacement patterns fairly close to the ones usually observed at failure in a general instability mode. The radial stiffness obtained by the preceding analysis may be expressed in the form

$$\alpha = K_3 (EI_b/D^3) \tag{4}$$

where E is the elastic modulus, I_b the moment of inertia of the frame cross section, and K_3 a constant. For a full circular frame $K_3 = 53.69$.

Criterion for General Instability

Substitution from Eqs. (2) and (4) into Eq. (1) yields

$$\frac{EI_b}{MD^2/L} = c \tag{5}$$

where

$$c = K_1 K_2 / K_3 \tag{6}$$

In the case of a full circular frame, the coefficient c is identical to the one proposed by Shanley.[5] His choice of $c = 6.25 \times 10^{-5}$, after correlation with experimental data, represents the minimum level of frame stiffness at which the designer can confidently assume that general instability failure will not occur. Such an approach cannot be easily implemented for the general case of a frame with a cutout and reinforced edges, due to lack of an adequate experimental data basis and, of course, the many variables involved in such a study. However, an indirect assessment of the coefficient c for a frame with a cutout can be realized on the basis of Shanley's original correlation.

First, Eq. (6) is written for the two distinct cases of a full (superscript f) frame and an incomplete (superscript i) frame with a cutout, as

$$c^i = K_1 K_2^i / K_3^i \tag{7}$$

$$c^f = K_1 K_2^f / K_3^f = 6.25 \times 10^{-5} \tag{8}$$

It is important to note that the value of the coefficient K_1 is a constant which depends neither on the bending stiffness nor on the shape of the transverse frame. Furthermore, the failure mode of the basic model shown in Fig. 1 will always be the same regardless of the nature of the source of transverse stiffness, e.g., a full transverse frame or a frame with a cutout, provided, of course, that ade-

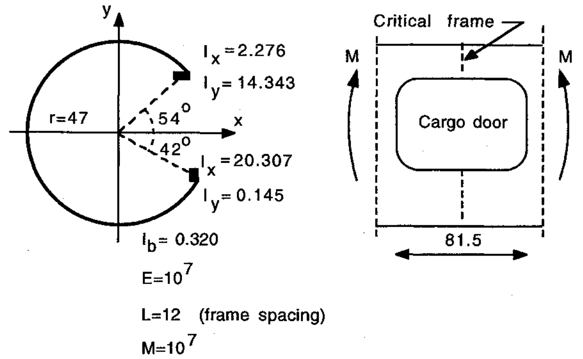

Fig. 4 Example problem.

quate lateral support is furnished by adjacent sections of the fuselage to prevent lateral deformations. This is true for any similar multi-degree-of-freedom spring-bar assembly as long as the transverse stiffnesses, provided by either type of framing, remain at comparable levels so that in either case the same mode of failure will be triggered by similar loading conditions. Therefore, in addition to being a constant, the coefficient K_1 must admit the same value in both Eqs. (7) and (8). Substitution from Eq. (3) into Eqs. (7) and (8) and elimination of K_1 yields

$$c^i = c^f \frac{K_3^f}{K_3^i} \frac{I^f}{I^i} \tag{9}$$

In deriving Eq. (9) it has been assumed that the angle φ of Eq. (3) is the same for the full and the incomplete frames. Based on the preceding discussion, the various terms appearing on the right-hand side of Eq. (9) are all known and, therefore, the coefficient c^i is an easily computed constant which, similarly to Shanley's c^f for full frames, represents the minimum level of stiffness for a frame with a cutout at which the possibility for general instability failure can be confidently ignored. Simply stated, Eq. (9) implies that for the design of an incomplete frame with a cutout to be equivalent to that of a similar full frame, any losses in moment of inertia due to the cutout must be proportionally compensated by gains in bending stiffness which can be realized only by the addition of reinforcement, possibly at the edge of the cutout. On the basis of Eqs. (5) and (9), and the preceding discussion, the proposed general instability criterion for frames with or without cutouts can be summarized in the following formula:

$$\frac{EI_b}{MD^2/L} \geq c^f \frac{K_3^f}{K_3^i} \frac{I^f}{I^i} \tag{10}$$

The basic assumption underlying the criterion just given is that the general instability mode of failure of a fuselage remains the same regardless of whether or not a cutout is present. This assumption is supported by experimental and analytical investigations on cylindrical shells[12] and is expected to be particularly true for usual fuselage structures where the edges of a cutout are always substantially reinforced due to other structural requirements. Apparently, in the special case of a complete circular frame, Eq. (10) reverts back to Shanley's criterion.

Numerical Example

The criterion developed in the preceding section is applied to the example shown in Fig. 4, for which any system of self-consistent units may be used. For illustrative purposes, the calculations for this particular example are based on the following simplifying, but conservative, assumptions: 1) only the ring frame over the centerline of the cargo door is considered critical, 2) any restraining effect the adjacent frames may have on the top and bottom edge stiffeners is ignored, 3) fixed-end conditions are assumed for the upper and lower edge stiffeners at either side of the cargo door, and 4) the torsional stiffness of both edge stiffeners is neglected. Based on these assumptions the flexural stiffnesses of the upper (superscript u) and lower (superscript l) stiffeners can be computed as $k_x^u = 50{,}870.78$, $k_y^u = 8{,}072.36$, $k_x^l = 514.28$, and $k_y^l = 72{,}023.49$. Using any standard finite element method (FEM) code the deformation of the vertical diameter under a pair of equal and opposite unit loads is approximately 4.065×10^{-3}, and thus from Eq. (5), $K_3^i = 63.85$. Direct substitution of these values into Eq. (10) yields

$$\frac{EI_b}{MD^2/L} = 4.35 \times 10^{-4} \geq c^f \frac{K_3^f}{K_3^i} \frac{I^f}{I^i} = 5.92 \times 10^{-5}$$

Therefore, the proposed design passes the proposed general instability criterion.

As an alternative example, consider the same basic design shown in Fig. 4 but with no edge stiffeners. The deformation of the vertical diameter under a pair of opposite unit loads is 5.093×10^{-2}, $K_3^i = 5.10$, and the right-hand side of Eq. (10) yields the value 7.41×10^{-4}. Comparison with the left-hand side reveals that this alternative design does not pass the proposed general instability criterion. It should be noted that the proximity of the values computed for the left- and right-hand sides of Eq. (10) for this particular example suggests, as it is intuitively expected, that even a minor edge reinforcement would suffice to satisfy the proposed criterion. However, this example is of only academic interest since a stiffened shell with a large unreinforced cutout would, most likely, fail due to local instability at the edge of the cutout.

With regard to the suitability of the preceding basic design for other parts of the fuselage with no cutouts, one could easily conclude that since the left-hand side of Eq. (10) is greater than Shanley's coefficient $c^f = 6.25 \times 10^{-5}$, this design also passes the general instability criterion for the specified load conditions.

Conclusions

A simple analytical method is presented for the prediction of general instability failure of stiffened cylindrical shells under pure bending conditions. The proposed formulation extends Shanley's original criterion[5] to stiffened shells with cutouts and edge reinforcement. But whereas Shanley's criterion is the product of a direct correlation of test data and a simple model, the proposed criterion is only indirectly correlated to the same test data via Shanley's coefficient c^f. Additional correlations with data obtained from tests of contemporary fuselage designs with cutouts would be necessary, if such data become available, for a more complete validation of the proposed criterion. However, in lieu of such data, and in view of experimental and numerical evidence that edge reinforcement in the perimeter of fuselage cutouts causes general instability modes of failure similar to the ones observed in fuselages with no cutouts, the proposed criterion can be used for design and weight calculations in absence of a rigorous FEM analysis.

References

[1]Becker, H., "Handbook of Structural Stability Part VI—Strength of Stiffened Curved Plates and Shells," NACA TN 3786, July 1958.

[2]Bruhn, E. F., *Analysis and Design of Flight Vehicle Structures*, Tri-State Offset, Cincinnati, OH, 1965.

[3]Langley Research Center, "Collected Papers on Instability of Shell Structures—1962," NASA TN D-1510, 1962.

[4]Bushnell, D., *Computerized Buckling Analysis of Shells*, Martinus Nijhoff, Dordrecht, The Netherlands, 1985.

[5]Shanley, F. R., "Simplified Analysis of General Instability of Stiffened Shells in Pure Bending," *Journal of Aeronautical Sciences*, Vol. 16, No. 10, 1949, pp. 590–592.

[6]Gerard, G., *Minimum Weight Analysis of Compression Structures*, New York Univ. Press, New York, 1956.

[7]Hoff, N. J., "General Instability of Monocoque Cylinders," *Journal of Aeronautical Sciences*, Vol. 10, No. 4, 1943, pp. 105–114, 130.

[8]Block, D. L., "Influence of Ring Stiffeners on Instability of Orthotropic Cylinders in Axial Compression," NASA TN D-2482, 1964.

[9]Hedgepeth, J. M., and Hall, D. B., "Stability of Stiffened Cylinders," AIAA Paper 65-79, Jan. 1965.

[10]Hoff, N. J., Boley, B. A., and Nardo, S. V., "The Inward Bulge Type Buckling of Monocoque Cylinders Subjected to Pure Bending," NACA TN 1499, 1948.

[11]Timoshenko, S. P., and Gere, J. M., *Theory of Elastic Stability*, McGraw-Hill, New York, 1961.

[12]Almroth, B. O., and Holmes, A. M. C., "Buckling of Shells with Cutouts, Experiment and Analysis," *International Journal of Solids and Structures*, Vol. 8, 1972, pp. 1057–1071.

Use of Mode Localization in Passive Control of Structural Buckling

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I. Introduction

ANDERSON localization, a phenomenon that was observed first in the field of solid state physics, has recently been given a lot of attention by structural dynamicists. Although the presence of irregularities in periodic lattices led to the localization of the electron eigenstates, the motion of a disordered elastic system was found to be confined to a limited part of the system. This study is an endeavor to introduce the mode localization phenomenon into the area of structural stability. By means of inducing deliberately some irregularities in the system, we may confine the structural buckling to only a limited part of the system. This process might be looked at as a passive control of buckling.

To cover the relevant literature in the area, we mention that the phenomenon of electron field localization in solid state physics[1] was extended to the field of structural dynamics by Hodge[2] and Hodge and Woodhouse.[3] Since then, the problem of mode localization has stimulated a number of structural dynamicists. Vibration localization in discrete mechanical systems has been considered by Pierre and Dowell,[4] Bendiksen,[5] Pierre,[6,7] and Wei and Pierre.[8] Vibration confinement in multispan beams has been considered by Pierre et al.,[9] Bouzit and Pierre,[10] and Lust et al.[11] Localization of normal modes of nonlinear systems has been recently considered by Vakakis.[12]

The applicability of the localization phenomenon to structural buckling was only addressed by Pierre and Plaut,[13] who examined the occurrence of buckling pattern localization in a two-span disordered column.

In the present Note, the effect of mode localization in a nearly periodic multispan column is studied using an exact formulation with the aid of the transfer matrix method. Mode localization is achieved by imposing specific types of irregularities, such as the inclusion of intermediate torsional springs and/or displacing intermediate supports. The buckled mode shapes indicate quite a strong confinement of the buckling to a fraction of the column.

II. Problem Formulation

We consider a multispan column of length $L\ (=\Sigma_{j=1}^{N}\ell_j)$, where the index j refers to a single span and N is the total number of spans. The panel is simply supported at both ends of each span. Moreover, identical torsional springs, each of stiffness c, are placed at the interior supports which exert restoring moments at these locations.

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The governing equation of the displacement of the individual panel is

$$\frac{\mathrm{d}^4 w_j}{\mathrm{d}x_j^4} + k^2\frac{\mathrm{d}^2 w_j}{\mathrm{d}x_j^2} = 0, \quad \text{in} \quad 0 < x_j < \ell_j \tag{1}$$

where w_j is the transverse displacement, $k^2 = P/EI$, P is the buckling axial load, and EI is the bending stiffness of the material. The general solution of Eq. (1) is

$$w_j = A_j\sin(kx_j) + B_j\cos(kx_j) + C_jx_j + D_j \tag{2}$$

where A_j, B_j, C_j, and D_j are arbitrary constants.

The boundary conditions at the ends of the panel element are

$$w_j(0) = w_j(\ell_j) = 0 \tag{3}$$

$$\frac{\mathrm{d}w_j}{\mathrm{d}x_j}(0) = \theta_{L_j}, \qquad \frac{\mathrm{d}w_j}{\mathrm{d}x_j}(\ell_j) = \theta_{R_j} \tag{4}$$

$$EI\frac{\mathrm{d}^2 w_j}{\mathrm{d}x_j^2}(0) + (1/2)c\theta_{L_j} = \hat{M}_{L_j}$$
$$-EI\frac{\mathrm{d}^2 w_j}{\mathrm{d}x_j^2}(\ell_j) + (1/2)c\theta_{R_j} = \hat{M}_{R_j} \tag{5}$$

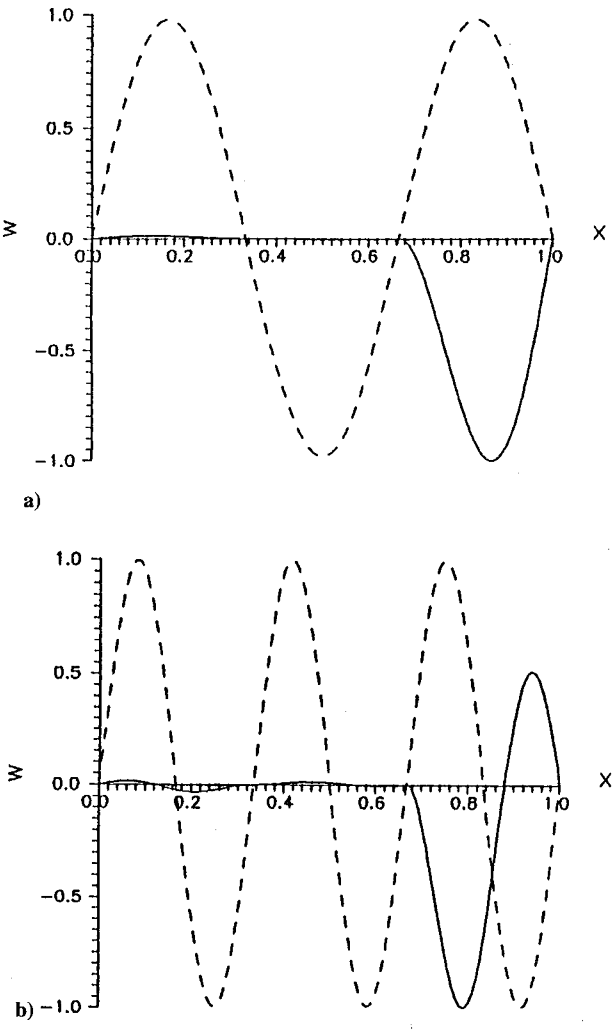

Fig. 1 Three-span column: a) first mode shape and b) second mode shape; ordered (dashed line) and disordered (solid line).